THEORY OF THE TRANSFORMER FOR THE NEUTRALIZING OF POWER INDUCTION IN TELEGRAPH CIRCUITS

BY

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THESIS

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I HEREBY RECOMMEND THAT THE THESIS PREPARED UNDER MY SUPERVISION BY William Riga Lyon

ENTITLED Theory of the Transformer for the Neutralizing of Power Induction in Telegraph Circuits

BE ACCEPTED AS FULFILLING THIS PART OF THE REQUIREMENTS FOR THE DEGREE OF Master of Science in Electrical Engineering.

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Recommendation concurred in:*

Committee on Final Examination*

*Required for doctor's degree but not for master's.

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THEORY OF THE TRANSFORMER FOR THE NEUTRALIZING OF POWER INDUCTION IN TELEGRAPH CIRCUITS

I

INTRODUCTION

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1. Power Line Induction

When two conductors forming a single phase alternating current line are connected to a source of high alternating potential, this line has the property of being capable of producing a disturbance in an adjacent neighboring closed circuit, when this closed circuit is NOT placed symetrically with respect to the power circuit as in Fig. 1.

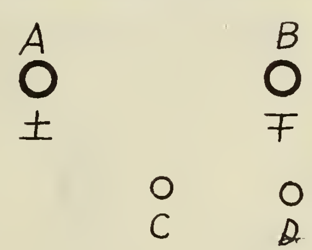

Fig. 1 Section View of a Power Line and a Communication Line

For instance, consider AB to be a single phase line at a high potential, and CD to be a telephone or telegraph line. The line AB causes two kinds of interference in the line CD, namely electrostatic and, when there is a load on the line or when the line is long enuf to have considerable capacity, there will also be electromagnetic induction

The conductor A produces an effect in both C and D, as does also conductor B.

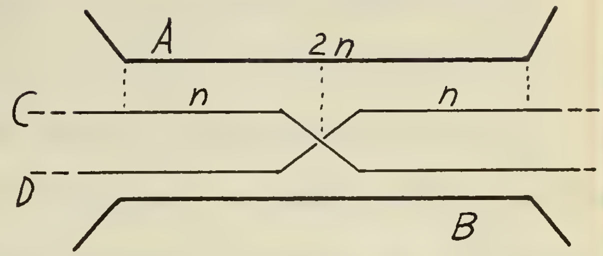

Fig. 2 Longitudinal View of of a Power Line and a Communication Line with Transposition

If the distance at which the line CD follows parallel to AB is considered as being "2n", and the wires CD be transposed at the distance "n", (see Fig. 2) it may be seen that both C and D affected by the same influences, precisely, and the resulting disturbances will be zero. If, as in the case of usual telegraph systems, there be but one metallic conductor C, the other conductor necessary for forming the closed circuit being the earth, it will be impossible to transpose the lines as in the case of two metallic wires. In this case, (see Fig. 3) the voltage induced must be eliminated by some other means than geometrcial transposition. One of the most frequently and easily applied means is by the use of the Neutraliz-Transformer.

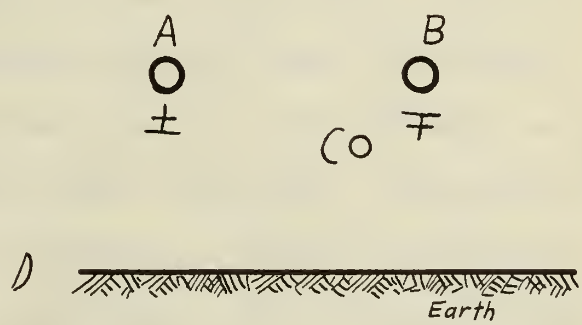

Fig. 3 Section View of Power Line and a Communication Line of the Single-conductor-ground-return type

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2. The Neutralizing Transformer Defined

The purpose of the neutralizing or compensating transformer is to secure a voltage equal in magnitude, but opposite in phase to

some electromotive force which it is desired to nullify. For example, everyone is familiar with the sight of telegraph lines which parallel railways some of which are electrified from an alternating current source. There are many places where telegraph lines traverse territory where, for a region of from two to forty miles, these lines will be paralleled by alternating current circuits as railways. in the case cited above, or transmission lines, as in Fig. 4.

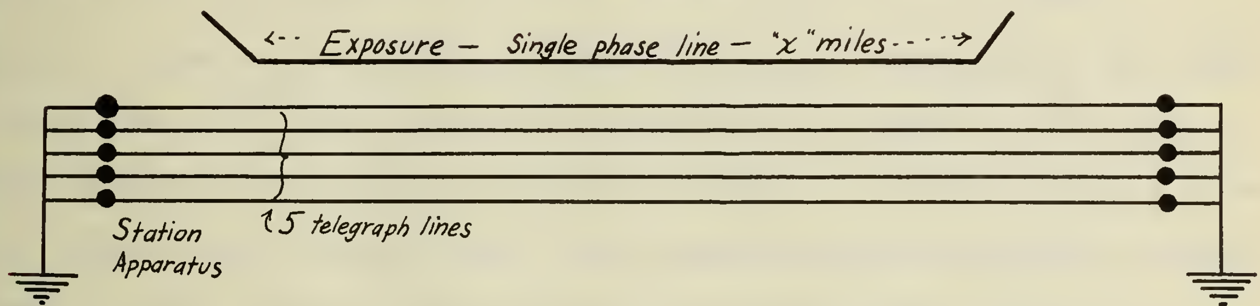

Fig. 4 Sketch Showing Telegraph Lines Being Paralleled by and Alternating Current Power Circuit.

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3. Applications of the Neutralizing Transformer

The proximity of this alternating current power for a long exposure, induces voltage in the telegraph lines, which interferes with the sending of messages. The only way for this voltage to be made nil is by the application of its exact opposite. Transposition of the telegraph wires would serve no purpose, because these circuits are of the one-wire-ground-return type. With these existing conditions, it is here that the sphere of this special type of transformer comes into prominence. The usual arrangement is to have a "primary line wire" DD (see Fig. 5) parallel the exposure. This wire is in series with the primary of the compensator and is also grounded at each end, thus forming the return circuit. The secondary coils,

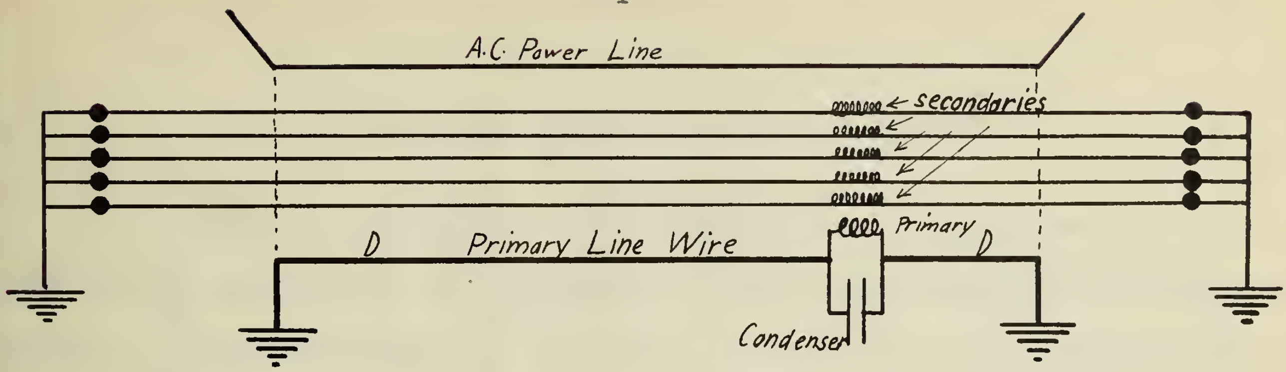

Fig. 5 Compensating Device Applied to Telegraph Circuits

equal in number to the lines which are to be compensated, are each in series with their respective circuits, as shown in the figure. The condenser in mulitple with the primary is selected with a view to making the voltage across the primary in phase with the voltage drop in the primary line wire and in the telegraph lines.

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4. Acknowledgements

The writer wishes to acknowledge the invaluable aid given by Mr. J. W. Milnor of the Western Union Telegraph Company, New York, under whom (with reference to another project) the initial stages of this work were done. The writer is indebted to Profesor E. B. Paine, Head of the Electrical Engineering Department of the University of Illinois who has had the general cupervision and guidance of this work under his care, and to the other members of the Department for many valuable suggestions. Particular mention is due in this respect to Profesor Morgan Brooks who has helped in making the subject matter clearer and to Mr. W. D. Cannon who has read the proof of the manuscript.

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5. Statement of the General Problem

It is found, in general, that Neutralizing Transformers do not completely neutralize or compensate; hence there remains a resultant voltage in the telegraph line, which, if it be of any considerable value, will cause an economic loss to the company operating the telegraph system, for it is obvious that signals cannot be transmitted as fast when there is interference, as when there is not. This interference or inductive disturbance is of two kinds:

(a) Interfercnee due to "residual" or uncompensated voltage, which is due to frequency variation of the disturbing power line from its normal operating frequency (that is, the "critical frequency" for which the resonant circuit of the compensator is adjusted). When this frequency is not that for which the primary of the Neutralizing Transformer and the shunt condenser was set to secure compensation, the secondary voltage will fail to completely compensate both by lack of being the proper phase and correct magnitude.

Fig. 6 Compensation at Critical Frequency

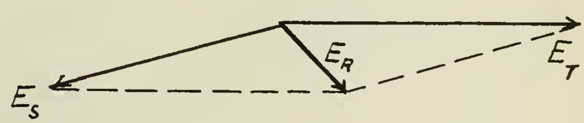

Fig. 7 "Residual Voltage" at some Other Frequency

(b) Interference due to induction between secondary coils, of which there is one for each telegraph line, producing what we have called a "mutual voltage". This induction is due to the telegraph currents

themselves, flowing thru inductively coupled circuits. The combined effect of these several lines is felt in each of them. The grouping of them is shown schematically by figure 8.

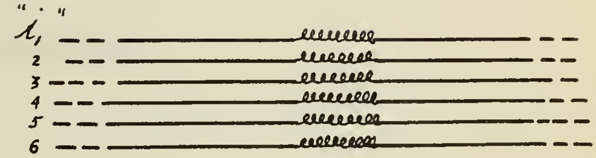

Fig. 8 Grouped Secondaries Producing "Mutual" Interference

The practical question, then, is: "How may the sum of the interferences be made as small as possible". This will be answered by an investigation into the general theory of operation of the compensator. Various circuits will be discussed and the relative advantages and disadvantages of each type will be pointed out.

II

THE GENERAL THEORY

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1. Operation at Critical Frequency

(a) Compensation of Power Line Induction

For this part of the problem, the chief interest is in securing voltage of the proper magnitued and of the right phase. With this in mine, the following considerations are offered.

Assume:

E_s Secondary e m f, equal and opposite to the total induced voltage from the power line for the exposure DD (see Fig. 5).

E_p Primary impressed e m f (see Fig. 9 for vector relation Fig. 10 for wiring diagram and Fig. 11 for detailed view of the neutralizer primary and shunt condenser.

R_p "Equivalent" (copper and iron) resistance of primary of transformer.

L_p Primary inductance.

Z_p Primary impedance.

C Capacity in parallel with primary.

Z_a Impedance of primary of transformer and condenser in multiple.

R_L Resistance of "primary line wire".

I_z Primary current

I_c Condenser current

I_L Total current

p Ratio of primary to secondary turns

φ Phase displacement between primary and line currents.

E_L Voltage across "primary line wire"
ω $2\pi f$, or 2. (frequency)

τ Time-constant L /R

Then, when neutralization takes place, we have, using the above nomenclature and the diagram given below:

1 $E_s = E_p + E_L$ vectorially and algebraically since all are in phase

2 $E_p = Z_p I_p$

3 $E_L = R_L I_L$

4 $E_s = \frac{E_P}{P} = E_P\left(\frac{N_s}{N_p}\right)$

5 $\frac{E_P}{P} = E_p + R_L I_L$ from 3 and 4

6 $\frac{Z_P I_P}{P} = Z_P I_P + R_L I_L$ from 2, and since

7 $I_P = \frac{I_L}{\cos\phi}$ see vector diagram

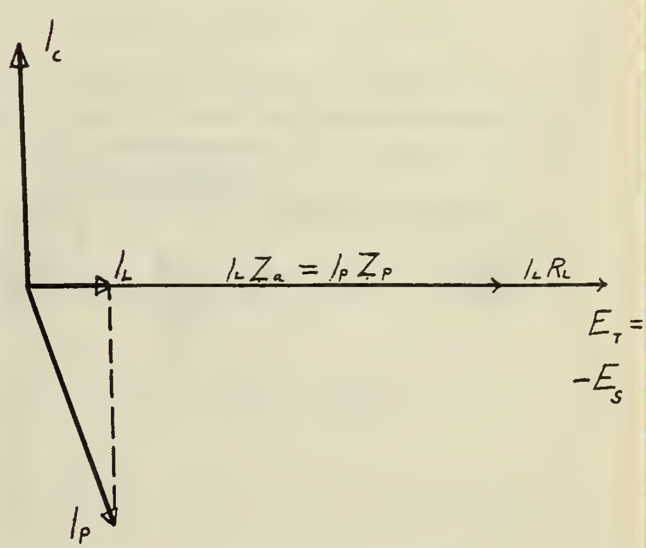
8 $\frac{Z_P}{P} \cdot \frac{I_L}{\cos\phi} = Z_P \cdot \frac{I_L}{\cos\phi} + R_L I_L$ from 6 and 7 and I being common and mulitplying by cos φ

9 $\frac{Z_P}{P} = Z_P + R_L \cos\phi$ on transforming

10 $Z_P\left(\frac{1}{P} - 1\right) = R_L \cos\phi$ which, on transposing the term

11 $Z_P = \frac{R_L P \cos\phi}{(1-P)}$ thus giving primary impedance in terms of primary line wire resistance, ratio of turns, and phase angle (hence, time-constant) without regard to the voltage and currents involved.

Fig. 9 Vector Diagram Showing Compensation of Power Line Induction



Having obtained an expression for the primary impedance, the next thing is to find the proper value of condenser to give the right phase relation. Proceeding as follows:

12	$\tau = \frac{L_p}{R_p}$	time-constant
13	$\tau\omega = \frac{L_p\omega}{R_p} = \frac{X_p}{R_p} = \tan\phi$	fundamental relations (see Fig. 12)
14	$\phi = \tan^{-1}(\tau\omega)$	from 13
15	$R_p = Z_p \sin\phi$	fundamental relations, as also
16	$L_p\omega = X_p = Z_p \sin\phi$	
17	$-jI_c X_c = jI_p Z_p$	from vector diagram of neutralizing transformer, also
18	$I_c = -I_p \sin\phi$	
19	$\therefore I_p \sin\phi \cdot X_c = I_p Z_p$	from 17 and 18
20	$\sin\phi \cdot X_c = Z_p$	by eliminating I ; but
21	$X_c = \frac{1}{C\omega}$	a fundamental relation
22	$\frac{\sin\phi}{C\omega} = Z_p$	from 20 and 21
23	$C = \frac{\sin\phi}{Z\omega}$	which is the theoretically correct value of capacity which will correct the phase.

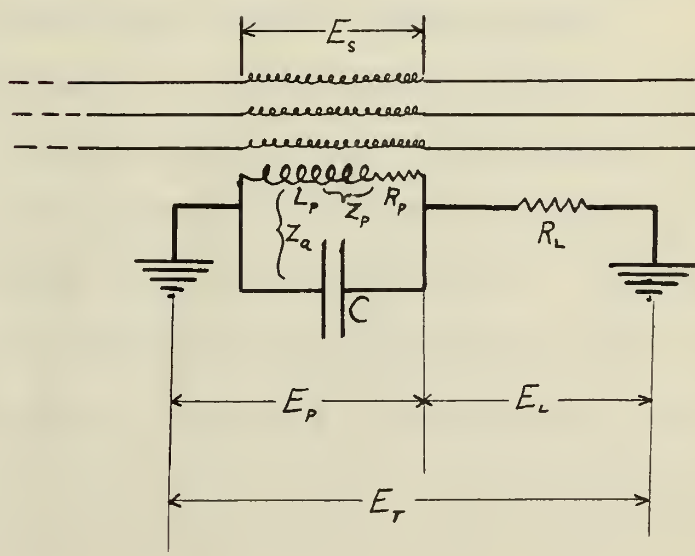


Fig. 10 Circuit Diagram for Compensation

Fig. 11 Circuit Diagram of Transformer Primary and Condenser

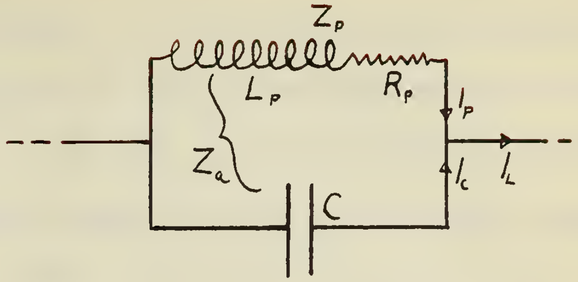
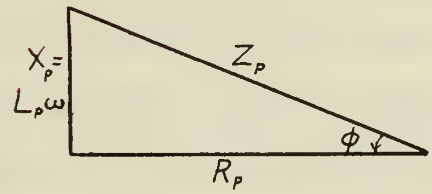


Fig. 12 Fundamental Vector Diagram



The significance of the above equations is as follows: Regardless of excitation current, the value of primary impedance is abso-;utely determined when the "primary line wire" resistance, ratio of turns and time-constant of the neutralizing transformer are known. It is customary in design to select some standard ratio of turns of primary to secondary, and some time-constant Data will be given later for indicating the most advantagous method of procedure in the selection of the proper values. Therefore, knowing the length of exposure to the alternating current power line, with the size of the compensating "primary line wire" to be used, we thereby predicate the primary impedance which will give correct compensation.

Having this value of impedance as a numerical quautity, we may calculate how much of this is resistance (copper and apparent iron resistance) and how much is reactance, with a view to altering the time-constant, whould the values seem disproportionate. The question of efficiency in the matter of Neutralizing Transformer losses is not paramount, since we are merely looking for compensation, and compensation not only for critical frequency at a predetermined in-induced voltage, but correction for deviations in the periodicity and

in the magnitude of that induced voltage. This is a radical departure from the usual type of transformer in which the impressed voltage is constant to within a few percent. In the case being dealt with here, the interference may vary from a small value at no load when atmospheric conditions are damp, to over 150 volts when the line current and conditions for electrostatic induction are most pronounced.

When this value of Υ has been selected, and ω has been fixed by the frequency of the line causing the disturbance which is to be compensated, the angle φ may be found from equation 14. R and L follow directly from equations 15 and 16. These formulas were of prime importance as a groundwork for subsequent investigations.

The foregoing quantities give us all the necessary data for calculating the correct capacity in equation 22. The functions below may be used to state our problem generally, thus

23 $Z_p = \vartheta\,(p, R_L, \Upsilon)$

24 $R_p = \sigma_1\,(Z_p, \Upsilon)$

25 $L_p = \sigma_2\,(Z_p, \Upsilon)$

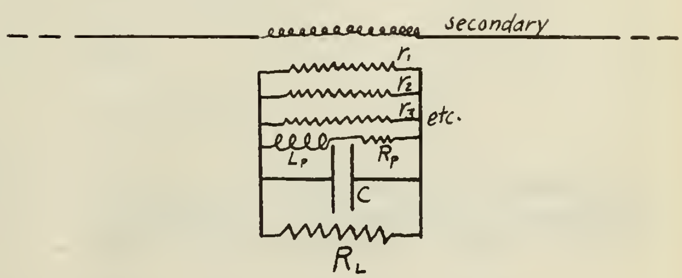
26 $C_p = \sigma_3\,(Z_p, \Upsilon)$

(b) Mutual Interference between Telegraph Wires

As has been stated before, the compensating voltage is applied to the telegraph line by virtue of the secondary coils of the neutralizer. These coils carry telegraph currents which are pulsating and in effect like alternating currents of a fluctuating number of cycles per second. While the relation at any moment of one signal current to any other is a random one, on the average, the effect of the interference from a number of circuits may be taken as a proportion of the interference which would take place with a lesser number

of lines, but all operating in unison. In practice, the average effect of 40 circuits operating is about the combined effect of 10; of 20, as 8; of 10, as 7; and of 6, as 5. We are furthermore assuming that the key frequency of the telegraph signals, is, on the whole that of the disturbing power line or railway. Mathematically, this quantity of disturbance may be obtained from the considerations given below (see Fig. 10). Any given secondary conductor has signal current flowing in it, and we may take this as approximately one-tenth ampere. The e m f which this particular secondary will induce in each of the others will be the product of the current of that secondary by the "equivalent impedance" of it. The equivalent impedance depends upon the ability to load the resulting primary circuit and the other secondaries, as the diagram (see Fig. 10) will show. For purposes of computation, the primary may be considered as being composed of three circuits in parallel: 1. Primary of transformer, 2. Condenser, 3. "Primary line wire". This latter branch is completed by the ground return. The secondary loadings will be the various circuits completed by their respective ground returns. The equivalent plan is shown by Fig. 13

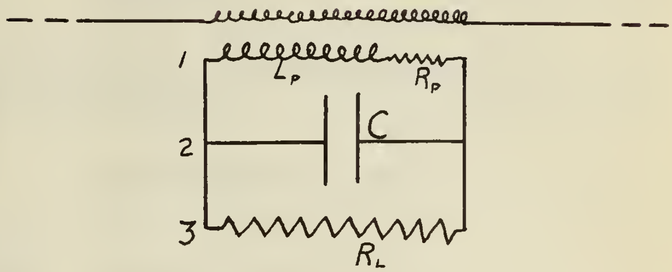
Fig. 13 Diagram of Equivalent Secondary Impedance of System



Owing to the fact that each telegraph circuit has a very high resistance which lies in the neighborhood of several thousand ohms, such that even the parallel arrangement of a considerable number would have a resistance which would be much greater than that of the primary line wire, we shall neglect the conductivity of these paths.

We are left, the, with the three parallel primary circuits, the diagra, being shown by Fig. 14

Fig. 14 Circuit of Approximate Equivalent Secondary Impedance



The following additional nomenclature is introduced at this point, the rest being used in the same sense as section (a).

Assume:

Y_1	Admittance of primary of neutralising transformer
Y_2	Susceptance of condenser
Y_3	Conductance of "Primary line wire"
Y_L	Total admittance of all three branches in parallel in terms of secondary admittance
Z_L	Total "Equivalent secondary impedance"
I_L'	Signal current in telegraph line
E_s'	Voltage induced in each secondary coil, due to impedance drop in secondary coil under discussion

Then:

27 $Z_p = R_p + jX = Z_p(\cos\phi + j\sin\phi)$ see Fig. 12

28 $X_c = \frac{1}{C\omega}$ and

29 $C = \frac{\sin\phi}{Z_p\omega}$ from 23

30 $R_L = \frac{Z_p(1-p)}{p\cos\phi}$ from 11. Having our separate elements, we may now consider our admittances, thus

31 $Y_L = p^2(Y_1 + Y_2 + Y_3)$ or, from 27, 29 and 30

32 $$Y_L = p^2\left[\frac{1}{Z_p(\cos\phi + j\sin\phi)} + j\frac{\sin\phi}{Z_p\,\omega}\,\omega + \frac{p\cos\phi}{Z_p(1-p)}\right]$$

factoring

33 $$Y_L = p^2\left[\frac{1}{Z_p(\cos\phi + j\sin\phi)} + \frac{1}{\frac{1}{jC\omega}} + \frac{1}{R_L}\right]$$

34 $$Y_\ell = \frac{p^2}{Z_p}\left[\frac{1}{\cos\phi + j\sin\phi} + j\sin\phi + \frac{p\cos\phi}{1-p}\right]$$

reducing to common denominator

35 $$Y_\ell = \frac{p^2}{Z_p}\left[\frac{1-p+j\sin\phi\cos\phi - \sin^2\phi - jp\sin\phi\cos\phi + p\sin^2\phi + p\cos^2\phi + jp\sin\phi\cos\phi}{(1-p)(\cos\phi + j\sin\phi)}\right]$$

collecting terms

36 $$Y_\ell = \frac{p^2}{Z_p}\left[\frac{1-p+j\sin\phi\cos\phi - \sin^2\phi + p}{(1-p)(\cos\phi + j\sin\phi)}\right]$$ simplifying

37 $$Y_\ell = \frac{p^2}{Z_p}\left[\frac{1+j\sin\phi\cos\phi - \sin^2\phi}{(1-p)(\cos\phi + j\sin\phi}\right]$$ trigonometric substitution

38 $$Y_\ell = \frac{p^2}{Z_p}\left[\frac{\cos^2\phi + j\sin\phi\cos\phi}{(1-p)(\cos\phi + j\sin\phi)}\right]$$ factoring

39 $$Y_\ell = \frac{p^2}{Z_p}\left[\frac{\cos\phi\,\cancel{(\cos\phi + j\sin\phi)}}{(1-p)(\cos\phi + j\sin\phi)}\right]$$ cancelling

40 $$Y_\ell = \frac{p^2}{Z_p}\,\frac{\cos\phi}{(1-p)}$$ from 11

41 $$Y_\ell = p^2\,\frac{1}{R_L\,p}$$ inverting

42 $$\therefore Z_\ell = \frac{R_L}{p}$$

This important formula tells us all there is to be known with reference to "equivalent secondary impedance" when the signal currents average the critical frequency (equation 42). Owing to the fact that this is a step-up transformer, "p" cannot possibly equal unity unless R is equal to zero, or Z equal to infinity, or unless the time-constant is infinite (see equation 11). Furthermore, it has been found from experience, that the best values of "p" are in the neighborhood of .9, because any excess of this makes a big increase in the cost of the transformer on account of the high time-constant required and is not warranted in the increased advantages of lower mutual impedance. Briefly, then, this equivalent impedance does not depend upon the constants of the transformer, except as regards the ratio of turns; but does depend upon the resistance of

of the primary line wire. It is therefore important to have the primary line wire low in resistance. The relationship of the proper size primary line wire will be more fully discussed in a following section (see chapter IV, section 2 A).

Having determined this equivalent impedance, the voltage caused by one circuit in each of the others is:

43 $E_s' = Z_L I_L'$ or finally, from 42 and 43

44 $E_s' = \frac{R_L I_L'}{P}$ volts per circuit.

The total effect of "n" circuits may be then ascertained from data given on page 12 near the top of the page.

These results are approximate, because we have assumed telegraph conductors of such high resistance as to have an inappreciable effect on the equivalent impedance of any secondary. Also this is approximate in the matter of the combining effect of several circuits, the telegraph signal current and the key frequency. The latter is not a fixed number, but depends upon the reat of transmitting and the characters being used.

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2. Operation at Non Critical Frequency

(a) Residual Voltage

When there is no variation in the magnitude of the induced voltage, but the frequency changes from the critical value for which the condenser was selected, there will be a "residual" or uncompensated voltage, (compare Figs. 6 and 7) which may be predetermined for any frequency, primary line wire resistance, ratio of turns and time-constant, which will be treated in the discussion following.

In addition to the nomenclature used above, the following new

terms will be introduced:

E_R "Residual" or uncompensated voltage

$\sim$ Frequency / 10

a(b) "a x 10^b"

45 $\dot{E}_R = \dot{E}_T - \dot{E}_S$ see Figs. 6 and 7

46 $E_T = E_p + E_L$

47 $E_p = Z_a I_L$ drop thru primary; now Z , the impedance of the parallel branch is found as follows

48 $Z_p = R_p + L_p\omega$ fundamental relations

49 $Y_p = \dfrac{1}{R_p + L_p\omega}$

50 $X_c = \dfrac{1}{-jC\omega} = \dfrac{j}{C\omega}$

51 $Y_c = jC\omega$ seperate admittances, having for a combined result

52 $Y_a = \dfrac{1}{R_p + jL\omega} + jC\omega$ and, on inverting

53 $Z_a = \dfrac{1}{\dfrac{1}{R_p + L_p\omega} + jC\omega}$ simplifying

54 $Z_a = \dfrac{R_p + jL_p\omega}{1 + C\omega R_p - C\omega L_p}$

55 $Z_a = \dfrac{R_p + jL_p\omega}{(1 - L_p\omega C) + jC\omega R_p}$ eliminating complex denominator

56 $Z_a = \dfrac{(R_p + jL_p\omega)(1 - L_pC\omega - jR_pC\omega)}{(1 - L_pC\omega)^2 + (jC\omega R_p)^2}$ collecting component terms

57 $Z_a = \dfrac{R_p + j[L\omega - R_p^2C\omega - CL_p^2\omega^3]}{[1 - CL_p\omega^2]^2 + (R_pC\omega)^2}$

58 $Z_a = \dfrac{R_p + j[L_p 2\pi (1) \sim - CR_p^2 2\pi (1) \sim - CL_p^2 8\pi^3 (3) \sim^3]}{[1 - CL_p 4\pi^2 (2) \sim^2]^2 + [CR\, 2\pi (2) \sim]^2}$

which is the form for rapid computation work

Thus, from R_p, L_p, C and $\sim$ we find the VECTORIAL expression for the impedance of the parallel circuit of the transformer primary

and the condenser. We then add this quantity VECTORIALLY to R which gives us the vectorial value of the total series impedance, Z . From Z , assuming the magnitude of the total interfering voltage to be 100, we may find I . Knowing I and the impedance of each section of the series parts, we may find the vector value of the voltage across the primary. This may be seen from the equation given below.

59 $100 = \dot{Z}_T \dot{I}_L = [\dot{Z}_a + R_L]\dot{I}_L$ and

60 $\dot{E}_p = \dot{Z}_a \dot{I}_L$ knowing the ratio of turns, we have E

61 $\dot{E}_s = \frac{\dot{E}_p}{P} = \frac{\dot{Z}_a \dot{I}_L}{P}$ which enables us to find E as a percentage of 100 from equation 45

We have made extensive use of equations 48 and following in the determination of data for the relation of the residual voltage to primary impedances for various time-constants (see Curve IV at the end of this work).

. . . .

(b) Mutual Interference

When the signals are not being transmitted with the same frequency as the critical one to which the resonant circuit is set, the expression for the "Mutual Interfering Voltage" becomes more complex. Instead of having the equivalent circuit composed of the transformer primary and shunt condenser of extreemly high impedance, this quantity is much lowered, particularly if the resonance be sharp. The reader is again referred to Fig. 14 with the mention that the solution is similar to that given for critical frequency, except that no value is taken for that frequency, that is, this case is general. As in the preceding, the total admittance is the sum of the separate admittances, thus:

62 $$Y_\ell = p^2\left[\frac{1}{R_p + jL_p\omega} + jC\omega + \frac{1}{R_L}\right]$$

which is the general form of 31; and reducing this to common denominator

63 $$Y_\ell = p^2\left[\frac{R_L + jC\omega R_L(R_p + jL_p\omega) + R_p + jL_p\omega}{R_L(R_p + jL_p\omega)}\right]$$

and we may find the total impedance

64 $$Z_\ell = \frac{1}{Y_\ell} = \frac{1}{p^2}\,\frac{R_L(R_p + L_p\omega)}{R_L + jC\omega R_L(R_p + jL_p\omega) + R_p + jL_p\omega}$$

collecting the "j" terms

65 $$Z_\ell = \frac{1}{p^2}\,\frac{R_L(R_p + jL_p\omega)}{(R_L + R_p - R_LCL_p\omega^2) + j(C\omega R_LR_p + L_p\omega)}$$

clearing complex terms from denominator

66 $$Z_\ell = \frac{1}{p^2}\,\frac{R_L(R_p + jL_p\omega)(R_L + R_p - R_LCL_p\omega^2 - j\{R_pR_LC\omega + L_p\omega\})}{(R_L + R_p - R_LCL_p\omega^2)^2 + (C\omega R_LR_p + L_p\omega)^2}$$

expanding

67 $$Z_\ell = \frac{1}{p^2}\,\frac{R_L[R_LR_p + R_p^2 - R_LR_pCL_p\omega^2 - jR_LR_pC\omega - jR_pL_p\omega + jR_pL_p\omega - jR_LCL_p^2\omega^3 + R_pR_LCL_p\omega^2 + L^2\omega^2]}{(R_L + R_p - R_LCL_p\omega^2)^2 + (R_LR_pC\omega + L_p\omega)^2}$$

cancelling like terms

68 $$Z_\ell = \frac{R_L}{p^2}\,\frac{[R_LR_p + R_p^2 - R_LR_p(L_p\omega^2 + R_LR_pCL_p\omega^2 + L_p^2\omega^2 + j\{-R_LR_p^2C\omega - R_pL_p\omega + R_LL_p\omega + R_pL_p\omega - R_LCL_p^2\omega^3\}]}{(R_L + R_p - R_LCL_p\omega^2)^2 + (R_pR_LC\omega + L_p\omega)^2}$$

rearranging

69 $$Z_\ell = \frac{R_L}{p^2}\,\frac{[R_LR_p + R_p^2 + L_p\omega^2 + j\{R_LL_p\omega - R_LR_p^2C\omega - R_LCL_p^2\omega^3\}]}{(R_L + R_p - R_LCL_p\omega^2)^2 + (R_pR_LC\omega + L_p\omega)^2}$$

and, in form for computing, similar to equation 58 in type

70 $$Z_\ell = \frac{R_L}{p^2}\,\frac{[R_LR_p + R_p^2 + L\,4\pi^2\,(2)\sim^2 + j\{R_LL_p\,2\pi\,(1)\sim - R_LR_p^2C\,2\pi\,(1)\sim - R_LCL_p^2\,8\pi^3\,(3)\sim^3\}]}{[R_p + R_L - R_LCL_p\,4\pi^2\,(2)\sim^2]^2 + [R_LR_pC\,2\pi\,(1)\sim + L_p\,2\pi\,(1)\sim]^2}$$

Having this equivalent secondary impedance, the mutual voltage is found as in the case for critical frequency, see page 14. This formula, like those from 58 to 61 inclusive, see page 17, was the basis for a set of curves (see Curve IV).

III

SPECIAL TYPES OF NEUTRALIZER

. . . .

INTRODUCTORY. Among the special types of compensating devices which occured to the writer, two will be mentioned, and a brief outline of the theory of each will be given. After the investigation was fairly started and the negative results of these forms forseen, the study was prosecuted vigorously with a view to proving mathematically the fact that the simple type would offer a minimum impedance hence a less interfering mutual voltage to the telegraph circuits.

. . . .

1. Parallel Compensators

In the effort to distribute the impedance by means of parallel branches, and in that way cause the total amount of the same to be the least, the following device was tried. Instead of having one core having all the secondary coils wound thereon, it was decided to study the effect of having as many small transformers as there were lines, the aim being to eliminate the inductive coupling between circuits. The total failure of such a scheme will be apparent.

(a) Compensation of Power Line Induction

This argument is very similar to that given on page 7ff, which see. Assume

R_s Resistance in series with each compensator

n Number of circuits

$\underline{R}$ "(R- nR)"

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

The arrangement of these parallel circuits is shown by Fig. 15

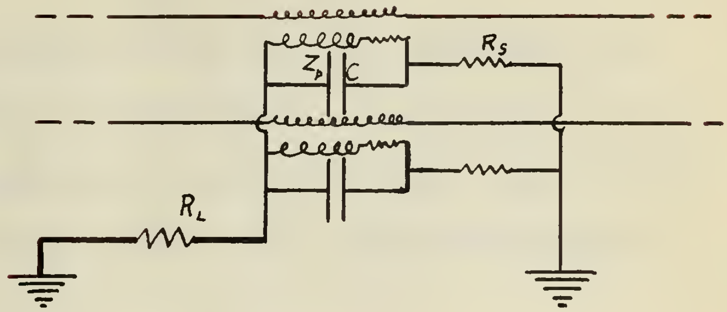


Fig. 15 Sketch Showing Parallel Compensators

It will also be noted that there must be as many individual condensers as there are neutralizing transformers. The manner in which this arrangement works out, when adjusted correctly and when the power line induction frequency is the "critical" frequency, may be seen from Fig. 16

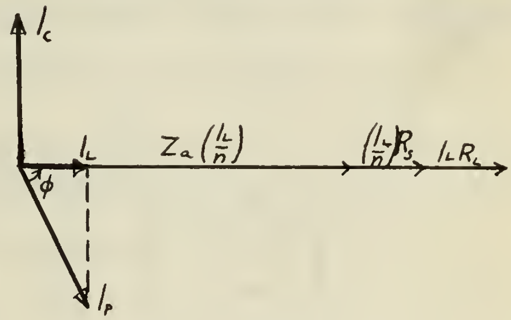


Fig. 16 Vector Diagram for Compensation with Parallel Arrangement

Referring to the nomenclature which has been used previously in addition to that given above (page 19):

71 $Z_a \frac{I_L}{n} + R_s \frac{I_L}{n} + R_L I_L = E_s = \frac{E_P}{P} = \frac{Z_a}{P}\,\frac{I_L}{n}$ — theory similar to that given in equations 1 to 11 inclusive

72 $\frac{Z_a I_L}{pn} = Z_a \frac{I_L}{n} + R_s \frac{I_L}{n} + R_L I_L$ — from 71; eliminating I

73 $\frac{Z_a}{pn} = \frac{Z_a}{n} + \frac{R_s}{n} + R_L$ — multiplying by "n"

74 $\frac{Z_a}{P} = Z_a + R_s + R_L n$ — factoring

75 $Z_a\left(\frac{1}{p} - 1\right) = R_s + n R_L$ — an expression for the impedance of each branch, but

76 $Z_a = \frac{(R_s + n R_L)P}{(1-P)}$

77 $Z_a = \frac{Z_p}{\cos\phi}$ Fig. 16 and equation 71

78 $Z_p = \frac{(R_s + nR_L)\, p \cos\phi}{(1-p)}$ from 76 and 77

This gives, it may be seen, a larger value for the individual impedances. Furthermore, in order to determine this quantity, it is necessary to assume two more factors, namely "n" (the number of circuits) and "R " (the resistance in series with each transformer.

(b) "Mutual Interfering Voltage"

The effect of this e m f is discussed below. It will be seen that metallic connection of the primaries plays a more detrimental role than simply magnetic coupling between the secondaries, when these are wound on the same core. The equivalent secondary impedance of any coil may be considered from its loading as in Fig. 17

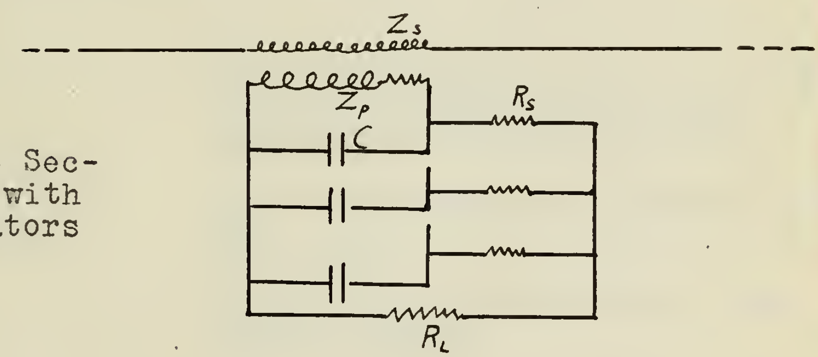


Fig. 17 Equivalent Secondary Impedance with Parallel Compensators

The argument is built from Fig. 14. At critical frequency, the potential drop across Z (that is, Z in parallel with C) is in phase with the vector sum of the currents of those two branches; therefore Z , taken as an entity, functions as pure resistance. Our diagram then becomes as in Fig. 18. When primaries are in parallel, the effect of the "nth" secondary on the other (n - 1), is found as follows. The other nomenclature used than that given below, is similar to that of the preceding sections:

R" Resistance of (n - 1) circuits in parallel

R"' R" in parallel with R

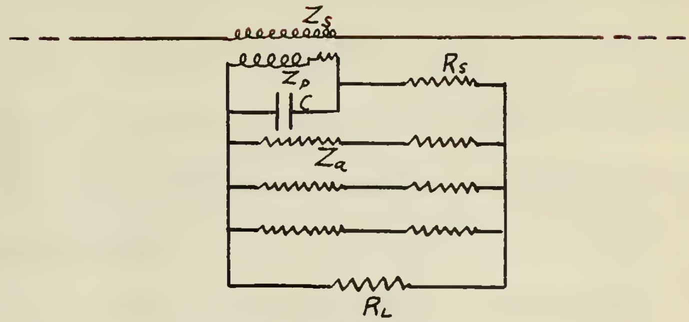


Fig. 18 Effective Resistance of Circuits Operating at Critical Frequency

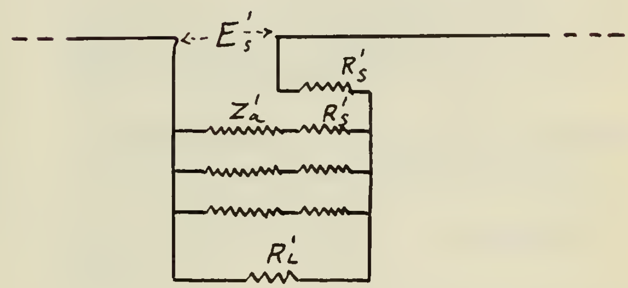


Fig. 19 Reduced Equivalent Diagram in terms of Secondary Values

79 $R'' = \frac{Z_a + R_s}{(n-1)}$ the resistance of (n - 1) circuits in parallel; and this quantity in parallel with R

80 $R''' = \frac{R'' R_L}{R'' + R_L}$ and this quantity in series with R

81 $R'''' = R'' + R_s$ which gives an equivalent secondary value

82 $Z_L = \frac{R''''}{P^2}$

This plan will not reduce the mutual interference, because, altho increasing R lessens the amount of disturbing current in the primary it at the same time increases the equivalent secondary impedance; consequently the potential drop across that impedance. This is because of the fact that the signal currents are adjusted to nearly constant value, regardless of the impedance to be overcome. Another point must be considered in the operation of this device (and which holds true for any compensator working on the resonance principle) is that the telegraph signals are sent with a varying rapidity; thus a vary-

ing equivalent secondary impedance is offered those signal currents because the impedance of a resonant circuit depends upon the frequency. This may offer a serious objection when the equivalent secondary impedance is relatively large and the resonance sharp, in that it will damp out signals of resonant frequency.

As an illustration, consider the inteference from a system of this type in the special case of R R , then

83 $$R'' = \frac{Z_a + R_L}{(n-1)}$$

now when Z is considered, it may be shown that its value is large compared with R

84 $$R'' = \frac{Z_a}{(n-1)}$$

is approximate; also the value (n - 1) is not from (n)

85 $$R'' = \frac{Z_a}{n}$$

and this quantity in parallel with R

86 $$R''' = \frac{R''R_L}{R''+R_L}$$

another working assumption which may be justified by experience is, when the number of lines is large Z /n = R

87 $$R''' = \frac{R_L^2}{2R_L} = \frac{R_L}{2}$$

and this in series with R = R

88 $$Z_p' = R'''' = \frac{R_L}{2} + R_L$$

which, in terms of secondary values, gives

89 $$Z_L = \frac{3R_L}{2p^2}$$

There is nothing mysterious in these assumptions, for the amount of error made by their use, is insignificant, also we may make Z almost any value we choose to come within the range of the case Z /n = R

In the simple circuit, see equation 42, we find the conditions much better than with the case taken above, thereby proving the advisability of abondoning the scheme just preceding. With the theoretical limit of workability, there would be 1.5 times the interference with this kind of neutralization as with the single core compensator.

. . . .

2. Compensation by Audion Bulbs

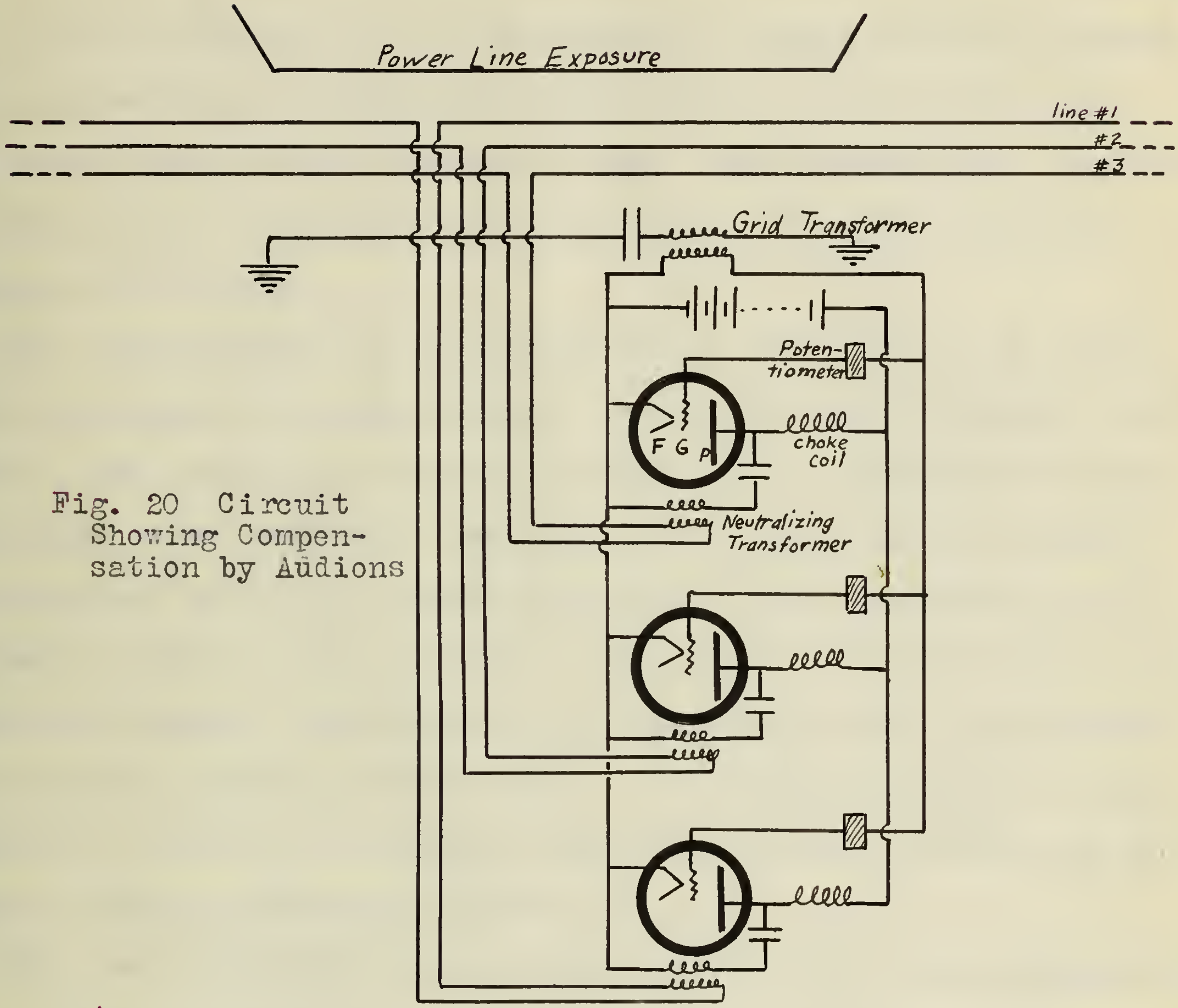


Fig. 20 Circuit Showing Compensation by Aüdions

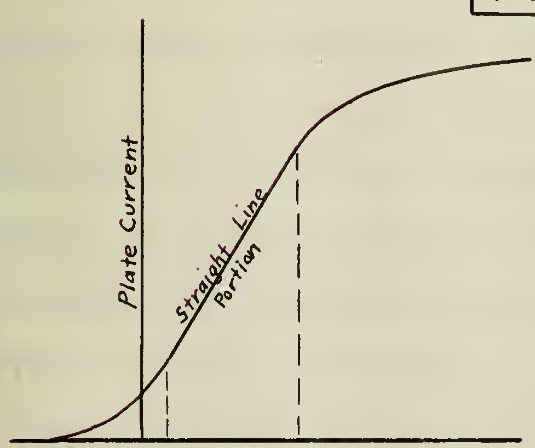


Fig. 21 Plate Current - Grid Voltage Characteristic of Audion

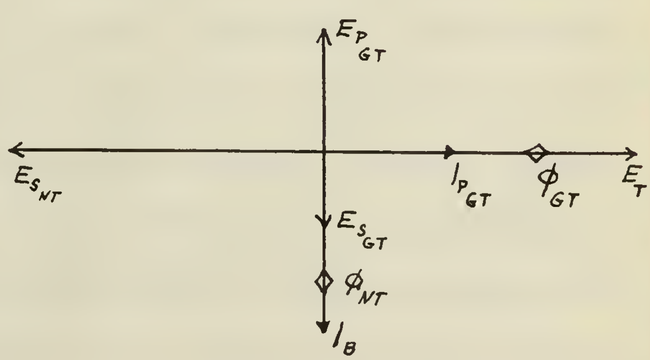


Fig. 22 Vector Diagram of Compensation

(a) Theory Applied to Correction of Interference

In this connection, it is necessary to examing what the factors for compensating at frequencies other than the critical one, are, and conditions modifying mutual voltage. The diagram given (see Fig. 20, shows three telegraph lines connected to the apparatus which operates as follows:

The compensating line being grounded at each end, is in series with the primary of a transformer. This is the grid potential transformer. The primary of this is in series with a condenser to secure the proper phase relation, (see Fig. 22 for vector relationships). The secondary potential secured by means of this transformer gives the same voltage to each filament - grid circuit of the several tubes. Since the filament - grid circuit is practically open (as regards the internal impedance of the tubes) the telegraph currents cannot "back-fire" into it and produce mutual interference, as was the case with all devices explained hitherto.

The same plate battery serves for all the tubes, the alternating current being excluded from these separate circuits by means of choke coils which offer at the same time, on account of their low copper resistances, very little obstacle to direct currents. In the same manner, the direct current is insulated from the compensating circuit by means of a condenser in series with that circuit asd set for resonance with the transformer at critical frequency. A potentiometer in each grid lead regulates the grid potential in such a manner that modulation takes place on the straight line portion of the "grid voltage - plate current characteristic (see Fig. 21). This being so, the plate current will flow thru the primary of the transformer circuit in quadrature with, and proportional to, the disturbing voltage.

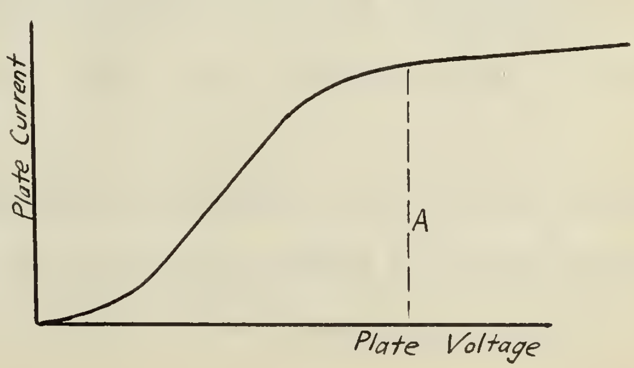
See Fig. 22. Since the condenser and primary of the grid transformer act much like an effective resistance if near the critical frequency , especially when the time-constant of that transformer is poor, we will have a current flowing thru that circuit (the primary line wire-primary condenser - primary of grid potential transformer) , which will be in phase with the e m f induced from the power line into the parallel exposed telegraph lines. The flux and current vectors (see Fig. 22) will be nearly coincident and the secondary voltage will be at a quadrature relation with the grid potential transformer flux, that is, roughly 90° from the power induction. But when we are working on the straight portion of the audion characteristic (see Fig. 21) pir tune current wo;; ne of the same phase as the grid voltage. This tube current is the one which operates the neutralizing transformer and is substantially in phase with the flux of that transformer, for the transformer is not delivering power and therefore does not need to furnish a component of primary current to take care of any secondary current, therefore the induced voltage to the secondary of the compensating transformer will lag behind the plate current by 90°, or the voltage induced by the power line by 180°.

If the time-constant of the neutralizing transformer by small, but more so on account of the high internal impedance of the tube, the resonance curve will be broad, and compensation will take place under quite a deviation from the critical frequency. This may be seen by considering a circuit of resistance, inductance and capacity in series (as the plate circuit, primary of transformer and condenser) This arrangement is shown by Fig. 20

(b) Results

It will be found in the use of tubes, however, that the economy is low, for the following reasons: (a) the plate current is small with a small bank of tubes, hence the ratio of turns of the neutralising transformer will have to be high in order to neutralize the secondary voltage. This would be satisfactory if this was the only thing which had to be done, but it is not, for telegraph currents must be practically unimpaired by this appended circuit. Now, with the large number of turns in the secondary which would be necessary, there would be heavy impedance to the secondary currents. This would be serious, but not fatal, for the impedance is nearly constant as regards the amount of current being transmitted. The difficulty which remains to be overcome is the electromotive force which would be induced into the primary of this compensator and thus function in conjunction with the plate battery. Altho the characteristic of this plate current - plate voltage curve is nearly horizontal, to the right of A (see Fig. 23), a difference of voltage of the order of one hundred or over, would take the plate current below that steady range, and thus affect the entire working an transformation ratios of the tube. The system must be one in which the neutralization does not interfere with the signal currents and vice versa. This may be done, as pointed out before, by a sufficiently large number of tubes to render the total impedance low; but from the tubes now available on the market, this would be economically out of the question.

Fig. 23 Plate Current Plate Voltage Characteristic



V

CONCLUSIONS

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1. General Observations

The operation and theory of the neutralizing transformer have been discussed from which the following facts have been noted:

A. The time-constant should be small to insure broadness of resonance for taking care of slight deviations from the critical frequency. This time-constant will vary somewhat on account of the variation of the impressed frequency. The explanation for this is as follows: The definition for the time-constant is L_p/R_p. Now, in our transformer, R is the effective value, and, as such has not only the copper component of resistance, but the "equivalent iron resistance" of hysterisis and eddy losses. These vary approximately with the square of the frequency, so that, when the range of period of the e m f which we have dealt with is large, there is an apparent change in the value of γ. This may be seen graphically from Curve I at the end of this thesis.

B. The compensator should have a characteristic such that it can readily accomodate itself to the wide range in impressed voltages to which is will be subjected. This is a special study in that connection.

C. In the case of mutual interference at critical frequency, the only way to reduce it is by having the ratio of turns as near the maximum as economy will permit, and at the same time reducing the

resistance of the primary line wire as much also as the economy will warrant (see Equation 11).

D. The method of parallel circuits, at best, does not give as good results as the simple general case first treated, for an improvement on one factor is accompanied by an overbalancing detriment in some other or others.

E. It is found in the case of vacuum tubes, that the cost of the tunes, the maintaining charges and other items do not offset the expense of a large size primary line wire. The chief obstacle is the high internal impedance of the bulbs and their very small output capacity.

. . . .

2. Some Applications of the Theory

We shall set forth briefly below, some of the practical results of this study on the Neutralizing of Power Induction.

A. Mention has been made of the "primary line wire", and of the desirability of having its resistance low (see equation 11). It was stated in section 1 (b) of chapter II, that current set up by "mutual interfering voltage" caused an economic loss to the interests operating the lines. This has been figured out in terms of interfering current. With this as a basis, the writer proceeded to balance this charge due to interference against that of the primary line wire, since it has been seen that a heavier primary line wire, by virtue of its lower resistance, will cause less interference than a smaller one would do.

It is reasonably safe to assume a resistance of about 2000 ohms for the average iron telegraph with which we are dealing; also a value

of 1/10 ampere for the message current. We have already shown that the mutual voltage is proportional to the equivalent secondary impedance and this is directly proportional to the resistance of the primary line wire (equation 11). Hence, we have a direct relation between the resistance of the primary line wire and the annual charges due to interference. We may establish another relation between the resistance of the wire to its length for a number of sizes and the same relation between annual charges to given length for the various sizes used. This will furnish a chart from which the proper size wire may be selected to give the minimum total annual charge for any given length of exposure (see chart - curve III).

Illustrative Example on the Use of Chart: Assume we have a twenty line system and that there is an exposure to alternating current power for a distance of twenty five miles. First, follow the horizontal line for "25 miles" on the chart, until intersects the line for 400000 circular mils both the right and the left, which, in the former case is the nearest line, while in the latter is the farthest one. At the point of intersection in each, respectively, follow the vertical to the horizontal axis on the left and determine the "Annual Charges for Line", which, it may be seen for 20 wires for 25 mile exposure, is $3700. The vertical on the right is followed until it intercepts the curve for "20 wires" (found in the first quadrant), from which point a horizontal course is taken until the "Annual Charges for Interference" axis is intersected, which, in this case, is at $800. Our total annual charges, then, using 400000 circular mils, is $4500. By trying other sizes, it will be found that #000 B. & S. gives the minimum total annual charge which is $3500. One also sees, owing to the fact that the lines in the two lower quadrants are straight lines and that those in the first have little curvature,

that nearly a constant size wire will give the most desirable results for ANY length exposure, it being merely a problem to fine out the size which takes care of any given number of telegraph lines best. A separate view of the "Interference Charges" to that of the "Resistance of the Primary Line Wire" is shown in Curve II at the end of this paper.

B. In the case of automatic sending apparatus, the frequency of the signals may, on the average, be assumed as 50 cycles. When we are compensating a power line, the normal frequency of which is 60 cycles, advantage may be taken of the difference of the telegraph operating frequency from the critical one, in the matter of resonance. What is desired is a broad resonance curve over the range in which the power frequency ordinarily varies, and a sharp curve for other frequencies, particularly 50 cycles. This sharp curve means a low equivalent secondary impedance, hence a low drop due to the telegraph currents, hence a low mutual voltage in the other secondaries of the transformer.

Certain relationships which alter the character of this resonance curve may be studied in the light of formulas 58 and 70. From these equations, we have the basis for plotting curves which will show us the most advantageous ratio of turns, time-constant, etc., for a given resistance of the primary line wire. A family of curves for residual voltage, mutual interfering voltage and one for the ratio of turns may be seen from Curve sheet IV. The more important characteristics of these curves will be pointed out. The most convenient abscissa to which to plot these curves was found to be the product of the primary impedance at critical frequency by the time-constant. Both kinds of interference are shown, with the ratio of turns for each, so it is simply a problem of selecting that sum which gives

the minimum total interference. Having decided on the value of Z, and p, we are ready for our design.

From the curves, it may be seen that:

1. E_R is nearly proportional to the frequency deviation, therefore, a 1/2 cycle variation either way from normal gives E_R half of the original value.

2. E_R is less with small time-constants.

3. E_R is less with high values of p.

4. E_R is NOT changed by R_p when Z_p is altered in the same proportion (see equation 58).

5. E_M is a function of the number of circuits exposed and to the transmitting current.

6. E_M is less with large time-constants

7. E_M is less with low values of p

8. E_M is DIRECTLY PROPORTIONAL to R_P when Z_p is altered in the same proportion (see equation 70).

. . . .

For carrying the work forward, the next step would involve trying out some spefic design. The laboratory model would be tested under conditions resembling, in so far as possible, those which the apparatus would be expected to remedy, and, if found satisfactory, would be installed in an operating line.

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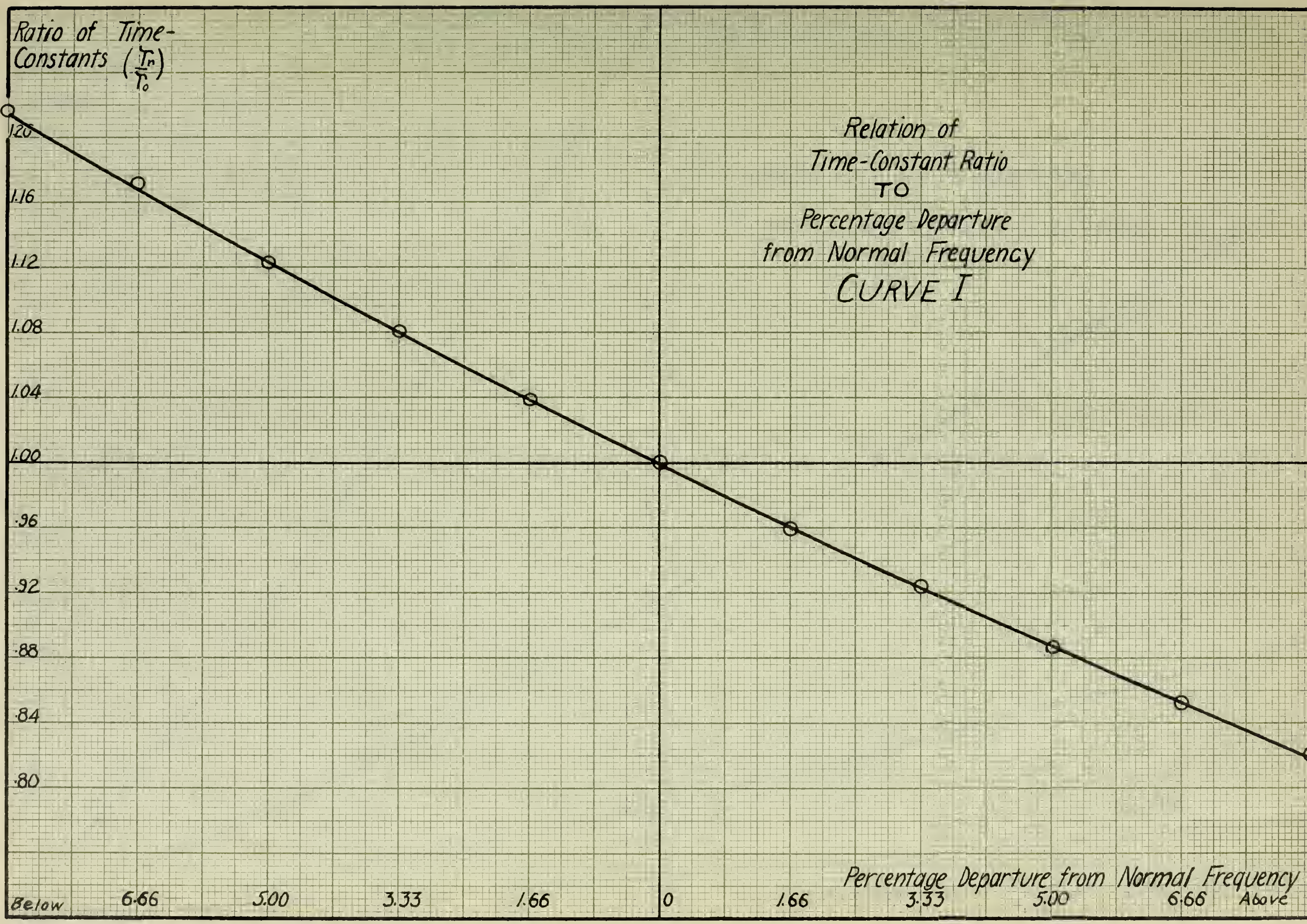
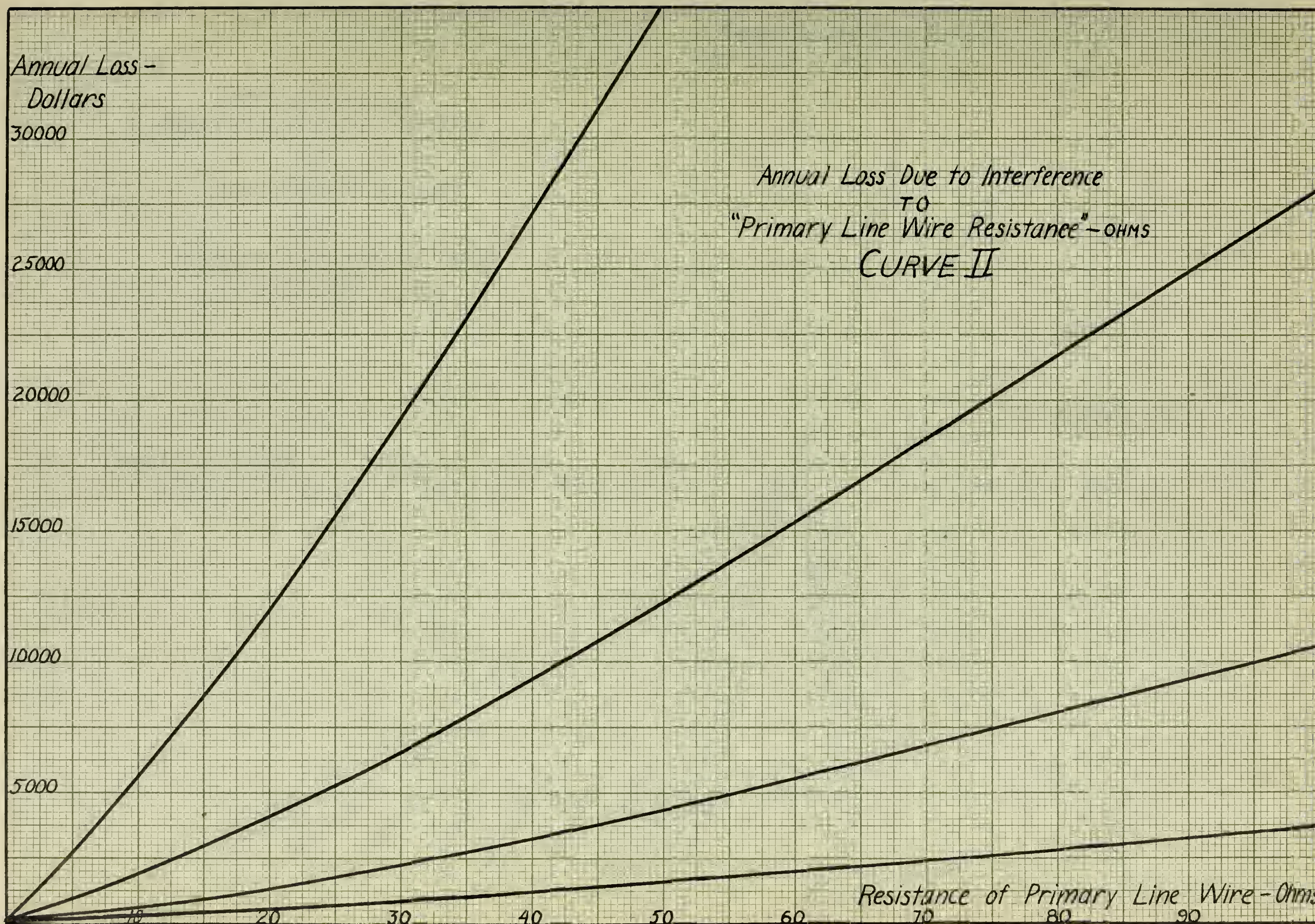

Ratio of Time-Constants $\left(\frac{T_n}{T_0}\right)$
Relation of
Time-Constant Ratio
TO
Percentage Departure
from Normal Frequency
CURVE I
1.20
1.16
1.12
1.08
1.04
1.00
.96
.92
.88
.84
.80
Percentage Departure from Normal Frequency
Below
6.66
5.00
3.33
1.66
0
1.66
3.33
5.00
6.66
Above


Annual Loss Due to Interference
TO
"Primary Line Wire Resistance"-OHMS
CURVE II
Annual Loss -
Dollars
30000
25000
20000
15000
10000
5000
Resistance of Primary Line Wire - Ohms
10
20
30
40
50
60
70
80
90

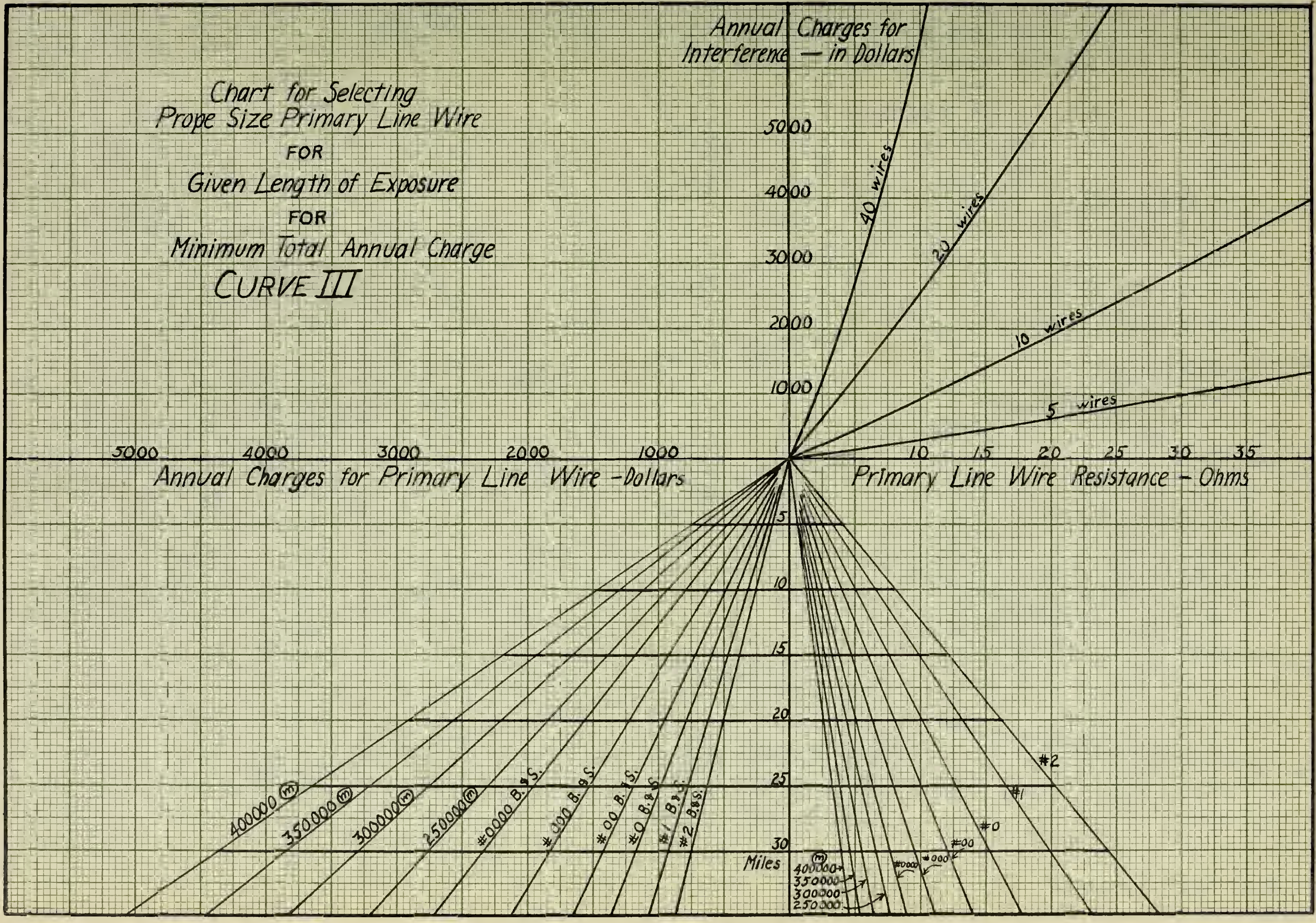


Chart for Selecting
Prope Size Primary Line Wire
FOR
Given Length of Exposure
FOR
Minimum Total Annual Charge
CURVE III
Annual Charges for
Interference — in Dollars
5000
4000
3000
2000
1000
40 wires
20 wires
10 wires
5 wires
5000
4000
3000
2000
1000
Annual Charges for Primary Line Wire -Dollars
10
15
20
25
30
35
Primary Line Wire Resistance - Ohms
5
10
15
20
25
30
Miles
400000 ⓜ
350000 ⓜ
300000 ⓜ
250000 ⓜ
#0000 B.&S.
#000 B.&S.
#00 B.&S.
#0 B.&S.
#1 B.&S.
#2 B.&S.
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#0
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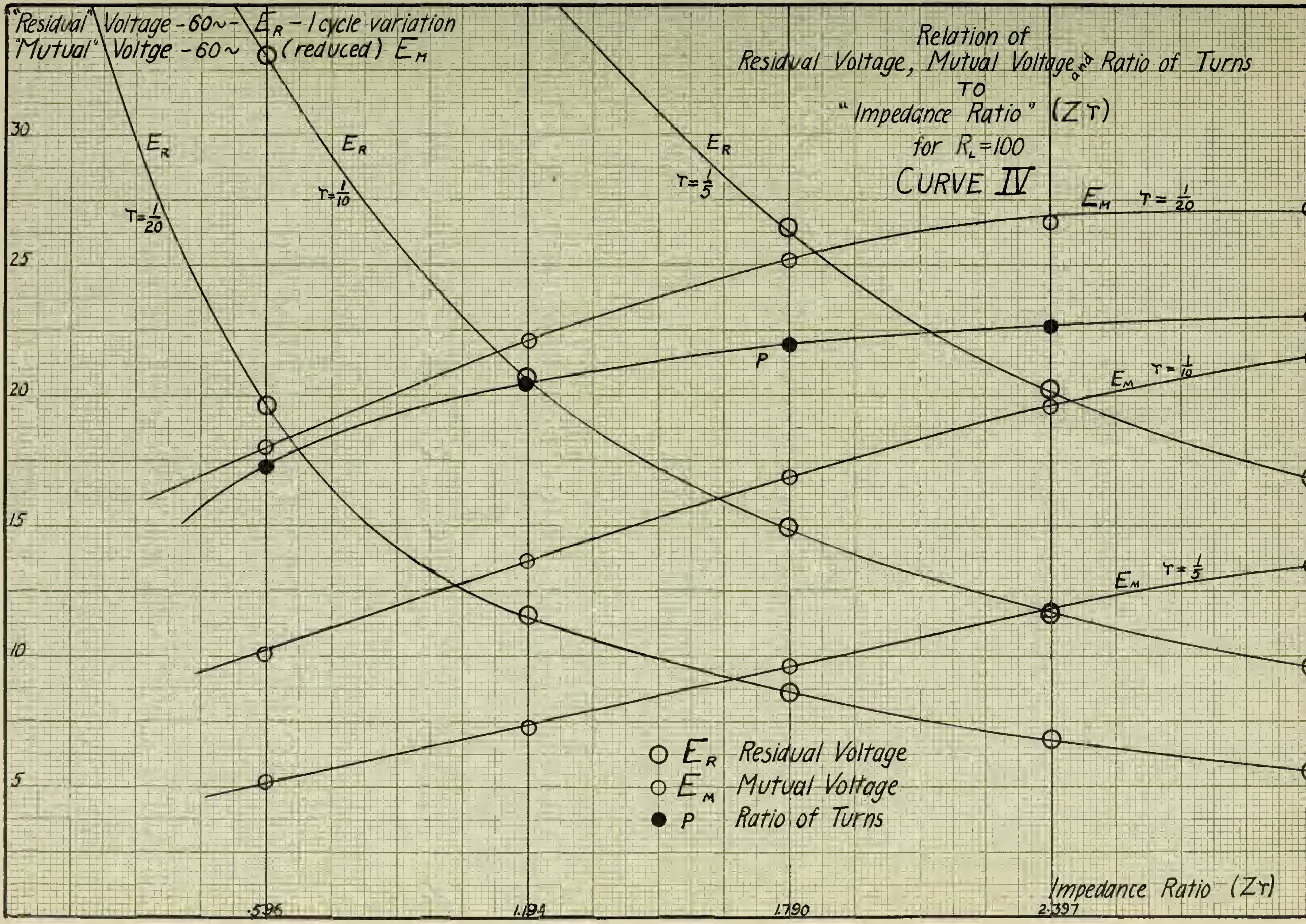
ⓜ
400000
350000
300000
250000


"Residual" Voltage - 60~ - E_R - 1 cycle variation
"Mutual" Voltge - 60~ (reduced) E_M
Relation of
Residual Voltage, Mutual Voltage and Ratio of Turns
TO
"Impedance Ratio" (Zr)
for $R_L=100$
CURVE IV
E_R
$r=\frac{1}{20}$
E_R
$r=\frac{1}{10}$
E_R
$r=\frac{1}{5}$
E_M $r=\frac{1}{20}$
E_M $r=\frac{1}{10}$
E_M $r=\frac{1}{5}$
P
30
25
20
15
10
5
.596
1.194
1.790
2.397
Impedance Ratio (Zr)
E_R Residual Voltage
E_M Mutual Voltage
P Ratio of Turns